Effect of Density Ratio on Binary Wing Flutter

Ll. T. Niblett*
Royal Aircraft Establishment, Farnborough, England

Introduction

By solving the binary flutter equations for an aircraft wing in a particular way, the overall effect of varying density ratio can be shown to be the sum of two distinct effects: one of these is exactly equivalent to that of varying reduced frequency; the other, although of overwhelming importance when the wing is light, is negligible when the wing is heavy. The particular form of solutions also illuminates the relationship between frequency coalescence and fuller flutter theories, as well as the conditions under which adding damping in one coordinate decreases rather than increases the flutter speed.

Solution of Flutter Equation

We write the flutter determinant as

$$\begin{vmatrix} -\sigma\omega^2c^4l+\rho c^2lV(i\omega cb_{11}+Vc_{11})+\sigma\omega_1^2c^4l & \rho c^2lV(i\omega cb_{12}+Vc_{12}) \\ \rho c^2lV(i\omega cb_{21}+Vc_{21}) & -\sigma\omega^2c^4l+\rho c^2lV(i\omega cb_{22}+Vc_{22})+\sigma\omega_2^2c^4l \end{vmatrix} \qquad (1)$$

where σ is the mass density of the wing, ω is frequency, c and l are typical lengths (chord and span), and so the first term of a typical element on the leading diagonal represents the structural inertia. ρ is the air density and V the airspeed, and the second term represents the complex aerodynamic forces that arise from simple harmonic motion in the generalized coordinates, with b_{rs} and c_{rs} functions of the reduced frequency of the oscillation. Because it is assumed that the coordinates are normal and have unit generalized mass, ω_1 and ω_2 are the still-air natural frequencies.

After making the substitutions $\rho V^2=\sigma c^2v^2$, $\mu=\sigma/\rho$, and $i\omega=\lambda$ and dividing each element by σc^4l the flutter equation can be written

$$\begin{vmatrix} \lambda^2+\lambda^{-1/2}b_{11}\nu\lambda+c_{11}\nu^2+\omega_1^2 & \mu^{-1/2}b_{12}\omega\lambda+c_{12}\nu^2 \\ \mu^{-1/2}b_{21}\nu\lambda+c_{21}\nu^2 & \lambda^2+\lambda^{-1/2}b_{22}\nu\lambda+c_{22}\nu^2+\omega_2^2 \end{vmatrix}=0 \qquad (2)$$

and its expansion

$$\lambda^4+p_1\lambda^3+p_2\lambda^2+p_3\lambda+p_4=0 \qquad (3)$$

where

$$p_1=\mu^{-1/2}v(b_{11}+b_{22})$$

$$p_2=(c_{11}+c_{22}+\mu^{-1}|B|)v^2+\omega_1^2+\omega_2^2$$

$$p_3=\mu^{-1/2}v\lambda(b,c)v^2+b_{11}\omega_2^2+b_{22}\omega_1^2)$$

$$p_4=|C|v^4+(c_{11}\omega_2^2+c_{22}\omega_1^2)v^2+\omega_1^2\omega_2^2$$

and

$$|B|=b_{11}b_{22}-b_{12}b_{21}, \quad (b,c)=b_{11}c_{22}+b_{22}c_{11}-b_{12}c_{21}-b_{21}c_{12}$$

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*Principal Scientific Officer, Materials and Structures Department.

At the critical flutter speed, Eq. (3) can be separated into real and imaginary parts

$$\lambda^4+p_2\lambda^2+p_4=0 \qquad (4a)$$

$$p_1\lambda^2+p_3=0 \qquad (4b)$$

Equation (4a) represents a conic in ω^2 and v^2

$$\omega^4-(c_{11}+c_{22}+\mu^{-1}|B|)\omega^2v^2+|C|v^4-(\omega_1^2+\omega_2^2)\omega^2 + (c_{11}\omega_2^2+c_{12}\omega_1^2)v^2+\omega_1^2\omega_2^2=0 \qquad (5a)$$

Eq. (4b) represents a straight line in ω^2 and v^2

$$(b_{11}+b_{22})\omega^2=(b,c)v^2+b_{11}\omega_2^2+b_{22}\omega_1^2 \qquad (5b)$$

and the critical flutter conditions are given by the points of intersection of the two.[1]

Analysis

The value of this form of solution is that the density ratio μ occurs explicitly only in the factor $(c_{11}+c_{22}+\mu^{-1}|B|)$. We can see immediately that when μ is large in value, its actual size has little effect on the critical values of ω^2 and v^2. Note, however, that although ω is the critical flutter frequency, the (true) critical speed is given by $V=\mu^{1/2}cv$ and, if μ is large, the reduced frequency, which is proportional to $\omega c/V$, is low. In the limit, b_{rs} and c_{rs} should be appropriate to a reduced frequency of zero.

Equation (5a) is the equation of a conic section. Here attention will be more or less limited to consideration of the hyperbola with center at positive v^2 because this is the most usual and interesting case.

The conic of Eq. (5a) intersects the ω^2 axis at ω_1^2 and ω_2^2. The straight line of Eq. (5b) intersects the ω^2 axis between ω_1^2 and ω_2^2 because b_{11} and b_{22}, must each be positive for the wing to be stable at low airspeeds.[1] The intersections of the axis can be by the same branch of the hyperbola (Fig. 1) or

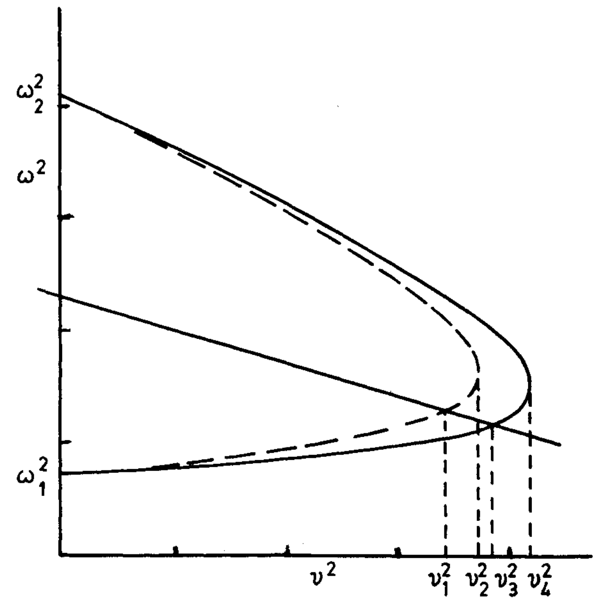

Fig. 1 Single-branch hyperbola.

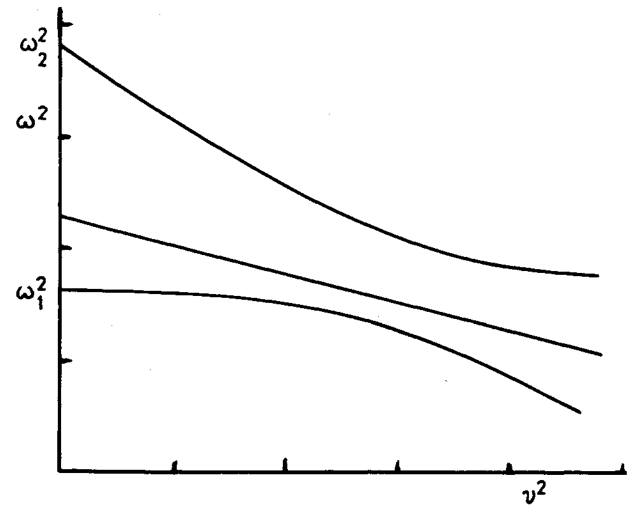

Fig. 2 Twin-branch hyperbola.

each branch (Fig. 2). If they are made by the same branch, and the center is at positive v^2, the wing must flutter at some value of v, but in the other case flutter depends on the orientation of the "damping" line [Eq. (5b)].

A condition that the intersections are made by the same branch is that Eq. (4a) has coincident roots λ^2 at some real value of ν^2. The condition for coincident roots is that

$$p_2^2 = 4p_4 \tag{6}$$

After substitution for the p_r and some reduction, Eq. (6) can be written, with $\bar{\omega} = \omega_1/\omega_2$ and $b = |B|/(1-\bar{\omega}^2)$,

$$\omega_2^2(1-\bar{\omega}^2)/\nu^2 = c_{11} - c_{22} - \lambda^4(1+\bar{\omega}^2)$$
$$\pm 2[-c_{12}c_{21} - \mu^1 b(c_{11} - c_{22}\bar{\omega}^2) + \mu^{-2}b^2\bar{\omega}^2]^{1/2} \tag{7}$$

The lower value of ν^2 given by Eq. (7) gives an upper limit to the lower critical speed because it is the largest value at which the damping line can intersect the conic (ν_4^2, Fig. 1). When $\mu \to \infty$, Eq. (7) tends to

$$\omega_2^2(1-\bar{\omega}^2)/\nu^2 = c_{11} - c_{22} \pm 2(-c_{12}c_{21})^{1/2} \tag{8}$$

which is the frequency-coalescence equation.[2] Hence, the speed given by frequency-coalescence theory can be regarded as an upper limit of flutter speed when the wing has infinite mass density (ν_2^2, Fig. 1). The full and dashed lines of Fig. 1 are typical of conics for sea-level and zero air densities.

If we take coordinate 1 to be flexure and coordinate 2 to be torsion, when $\omega_2^2 - \omega_1^2$, $c_{11} - c_{22}$, and $-c_{12}c_{21}$ will all be positive. This assumes that the wing is conventional, not mass-balanced, and neither swept forward nor of low aspect ratio. The associated conic will be an ellipse or a hyperbola according to the relative sizes of $c_{11} - c_{22}$ and $-c_{12}c_{21}$ but, in either case, there will be a lower flutter speed. Note that if $\omega_2^2 \gg \omega_1^2$,

$$\omega_2^2(1-\bar{\omega}^2)/\nu^2 \doteqdot \omega_2^2/\nu^2 = \sigma\omega_2^2 c^2/(\rho V^2) \tag{9}$$

$\sigma\omega_2^2 c^2$ is proportional to the torsional stiffness of the wing structure and hence, the square root of kinetic pressure divided by torsional stiffness provides a better flutter speed parameter than airspeed divided by the product of torsional frequency and chord.

An increase in density ratio lowers the smaller value of ν^2 (Ref. 1) but, with wings of current desities, not at an excessive rate. The common experience in the past, that the equivalent flutter speed drops slightly as height is increased ($\nu_3^2 \to \nu_1^2$, Fig. 1), confirms this and implies that the aerodynamic coefficients b_{rs} and c_{rs} do not vary overmuch with reduced frequency in this case.

When the density ratio is much lower, however, its effect on flutter is much more marked, for the limit of Eq. (7) as $\mu \to 0$ is

$$\omega_2^2(1-\bar{\omega}^2)/\nu^2 = -\mu^1\,|B|\,(1 \pm \bar{\omega})^2 \tag{10}$$

Both the values of ν^2 given by Eq. (10) are negative, so there is no coalescence-type flutter. The parts of the hyperbola in the quadrant of positive ν^2 and ω^2 are close to the asymptotes, and the presence or absence of flutter depends on the slope of the damping line. Experience shows that, generally, the slope is indeed between the slopes of the asymptotes and the wing is free of flutter. Wings of quite high density might also be free from coalescence flutter, however, because, when density ratio is reduced, a "Fig. 1" hyperbola first changes to one with separate branches intersecting the ω^2 axis (Fig. 2). The significant characteristics of such a hyperbola are very similar to those of the hyperbola with center at negative ν^2 described earlier. The hyperbola is of this type when its center is on the ω^2 axis.[1] From Eq. (7), the center is on this axis when

$$\mu = |B|(1+\bar{\omega}^2)/((c_{11} - c_{22})(1-\bar{\omega}^2)) \tag{11}$$

This is probably a lower limit of the value of μ at which flutter disappears. Note that the closer the frequencies of the normal modes, the higher this limit is. The foregoing explains the classic L shape of the curve of $\rho V^2/(\sigma\omega_2^2 c^2)$ against ρ^{-1}. The basic reason for the disappearance of flutter as density ratio is reduced is that, if equivalent airspeed is kept constant, the real parts of the aerodynamic forces are constant with ρ, but their imaginary parts increase as $\rho^{1/2}$, so that eventually the dissipative forces dominate. There is an additional effect beacuse the true airspeed is reduced as ρ is increased and, hence, if the flutter frequency remains more or less the same, the reduced frequency increases. This is likely to increase the stability of the system further—at least in the case of incompressible aerodynamics.[3]

The damping line has the same eqution, whatever the density ratio. Analysis of it has not proved very fruitful, but experience shows that the line is generally well behaved in the sense that its slope is between the slopes of the asymptotes of the conic and it intersects the conic toward the latter's nose.

It has often been observed that the flutter speed of a lightly damped wing is below its frequency-coalescence speed. The conditions necessary for this to happen can be visualized using Fig. 1. The frequency-coalescence speed is ν^2 at the apex of the dahed conic. If the in-quadrature forces are small, $|B|$ is very small, the conic is scarely different, and ν_2 is also a good approximation to the maximum possible flutter speed of the damped wing. The in-quadrature forces are unlikely to be in the proportions necessary to send the damping line through the apex of the conic, and the flutter speed will be less than ν_2 (say ν_1). Proportional increases in the in-quadrature forces increase $|B|$ and have the same effect as a decrease in μ, "expanding" the conic (to the full line, say), but not changing the damping line, so that the flutter speed is raised (to ν_3, say). Whether the raised flutter speed is higher than the frequency-coalescence speed cannot be predicted easily. As mentioned before, the dissipative action of the in-quadrature forces can be large enough for the hperbola to be one having two active branches with no frequency coalescence and probably no intersection with the damping line.

Another phenomenon sometimes observed when the density ratio is high is that the flutter speed can be reduced by increasing the damping in one of the degrees of freedom. From Eq. (5b), the intersection of the damping line and the ω^2 axis is where

$$(b_{11} + b_{22})\omega^2 = b_{11}\omega_2^2 + b_{22}\omega_1^2 \tag{12}$$

and so, if

$$b_{rr} \gg b_{ss}, \qquad \omega \doteq \omega_s \tag{13}$$

Thus, if the direct dampings are not balanced, the damping line is close to the conic at $\nu^2 = 0$, and its intersection with the conic probably occurs at a low value of ν^2. Decreasing the larger damping coefficient or increasing the smaller coefficient, while not altering the conic significantly because of the high density ratio, moves the intersection of the damping line and the axis away from the conic and, providing the change in the slope of the damping line is not excessive, either of these changes moves the intersection of damping line and conic to a higher value of ν^2 and, hence, increases the flutter speed. Qualitatively, it matters not whether the direct damping is aerodynamic or structural.

Concluding Remarks

It has been shown that the effect of density ratio on flutter can be clarified by a particular form of solution of the flutter equation that leads to the visualization of the solution as the intersection of a conic and a straight line. Part of the effect is surffered only by the conic. Strictly, the aerodynamic coefficients should always be those appropriate to the reduced frequency at the intersection, but experience shows that insights into the equtions can be obtained even when the coefficients are not exact. This has been demonstrated by the clarification of the conditions under which "anomalous" behaviors with increase of damping occur.

References

[1]Niblett, L. T., "A Graphical Representation of the Binary Flutter Equations in Normal Coordinates," ARC R&M 3496, 1966.

[2]Karman, T. V., and Biot, M. A., *Mathematical Methods in Engineering*, McGraw-Hill, New York, 1940.

[3]Lambourne, N. C., "On the Conditions under Which Energy Can Be Extracted from an Air Stream by an Oscillating Aerofoil," *The Aeronautical Quaterly*, Vol. 4, Aug. 1953, pp. 54–68.

Technical Comments

Comment on "The Role of Structural and Aerodynamic Damping on the Aeroelastic Behavior of Wings"

William P. Rodden*
La Cañada, Flintridge, California

AFTER Ref. 1 was presented at the Symposium on Structural Dynamics and Aeroelasticity in 1965, this commentator offered some remarks and showed two slides at the meeting that were related to the inaccuracies associated with the aerodynamic approximations employed. Since it is not an American custom to append remarks in meeting proceedings, the remarks and slides were not published. However, they have been included in this commentator's notes for teaching aeroelasticity.[2] Reference 3 now appears, 21 years later, and draws its conclusions based on similar aerodynamic approximations, again with no regard for the magnitude of the associated errors or the current state-of-the-art of unsteady aerodynamic theory. This suggests that a more formal publication of the 1965 remarks and slides is long overdue.

The first slide (Fig. 1) compared Pines' quasisteady aerodynamic method[4] with Theodorsen's exact aerodynamic solution[5] in a flutter analysis of a two-degree-of-freedom airfoil at sea level. The example airfoil was mounted at its 40% chord elastic axis on bending and torsion springs, having uncoupled bending and torsion frequencies of $\omega_h = 10$ rad/s and $\omega_\theta = 25$ rad/s, respectively. The airfoil mass m was such that its mass ratio was $\mu = m/\pi\rho b^2 = 20.0$, where ρ was the density at sea level and the semichord was $b = 3.0$ ft. A range of centroids was considered from 35 to 85% chord, assuming a constant dimensionless centroidal radius of gyration $r_c = 0.49$, or a dimensional radius of gyration of $r_c b = 1.47$ ft. Also shown in Fig. 1 is a curve based on the variation suggested by Pines in which the airfoil lift curve slope c_{ℓ_α} was refined (iteratively) by $c_{\ell_\alpha} = 2\pi|C(k)|$ in which $C(k) = F(k) + iG(k)$ is the Theodorsen function, $|C(k)|$ its magnitude, and $k = \omega b/V$ the reduced frequency. This refinement is seen to improve the agreement with the exact solution for a limited range of centroid (from 40 to 50% chord). More significantly, however, Pines' method predicts *no* flutter for centroids forward of the elastic axis (note that no coalescence can occur with a centroid forward of the elastic axis) and much higher flutter speeds than the exact solution for aft centroid locations (behind the 50% chord).

The second slide (Fig. 2) compared Dugundji's low-frequency aerodynamic method[6] with the Theodorsen solu-

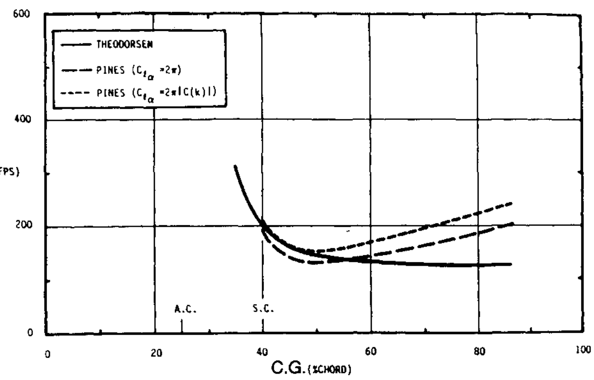

Fig. 1 Comparison of general unsteady and quasisteady flutter analyses.

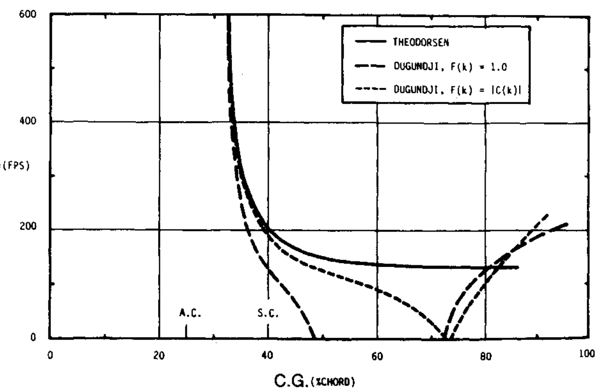

Fig. 2 Comparison of general unsteady and first-order unsteady flutter analyses.

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*Consulting Engineer. Fellow AIAA.